(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 Roll No. | | | | | | | | | | |

B.Tech.

(SEM II) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

MATHEMATICS - II

Time : 3 Hours] *[Total Marks : 100*

SECTION - A

Note : *Attempt **all** questions. Each question carries **equal** marks :* **(10x2=20)**

1. (a) The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 = 0$ are ...... and .....

(b) Pick the correct statement from the following :

(i) Integrating factor to a differential equation is unique.

(ii) $y = ex$ is the general solution of the $\frac{ydx - xdy}{x^2} = 0$.

(c) Write the Rodrigue formula $P_n(x) = ...$

(d) If J_0 and J_1 are Bessel's functions, then $J_1(x)$ is given by :

(i) $J_0(x) - \frac{1}{x}J_1(x)$ (ii) $J_0 + \frac{1}{x}J_1(x)$

(iii) $-J_0(x)$ (iv) None of these

(e) $L^{-1}\left\{\frac{1}{S^n}\right\}$ exist only when the value of n is :

(i) Negative integer (ii) Positive integer

(iii) Zero (iv) None of these

(f) Pick the correct statement for final value theorem of Laplace transform :

(i) $\underset{t\to 0}{\text{Lt}}\ f(t) = \underset{s\to\infty}{\text{Lt}}\ sF(s)$ (ii) $\underset{t\to\infty}{\text{Lt}}\ f(t) = \underset{s\to 0}{\text{Lt}}\ sF(s)$

(g) Fourier coefficient 'a_0' in Fourier series expansion of a function represents the :

(i) maximum value of the function (ii) mean value of the function

(iii) minimum value of the function (iv) none of these

(h) If the Fourier series of $f(x)$ has only cosine terms then $f(x)$ must be :
(i) odd function (ii) even function

(i) The PDE $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ is known as :
(i) wave equation (ii) heat equation
(iii) Laplace equation (iv) none of these

(j) The PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, is :
(i) parabolic (ii) elliptic (iii) hyperbolic (iv) circular

SECTION - B

Note : Attempt **any three** parts from this section. Each part carry equal marks : **(3x10=30)**

2. (a) A particle of mass m moves in a straight line under the action of force mn^2x which is always directed towards a fixed point O on the line. Determine the displacement $x(t)$ if the resistance to the motion is $2\,\lambda mnv$ given that initially $x=0$, $\frac{dx}{dt}=0$ $(0<\lambda<1)$.

(b) Using Frobenius method, obtain a series solution in powers of x for differential equation : $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$.

(c) Using Laplace transform, solve the differential equation :

$$\frac{d^2y}{dx^2} + n^2 y = a\sin(nx+2)$$

Given, $y(0)=0$ and $y'(0)=0$.

(d) Find the fourier series to represent the function $f(x)$ given by :

$$f(x) = \begin{cases} -K & \text{for } -\pi < x < 0 \\ K & \text{for } 0 < x < \pi \end{cases}$$

Hence show that :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}.$$

(e) Solve the Laplace's equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle in the xy-plane, $0 \le x \le a$ and $0 \le y \le b$ satisfying the following boundary conditions $u(x, 0)=0$, $u(x, b)=0$, $u(0, y)=0$ and $u(a, y)=f(y)$.

SECTION - C

Note : Attempt **any two** parts from all questions of this section. Each part carry equal marks :

(5x2x5=50)

3. (a) Solve the following differential equation :

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 3x + \cos 2x.$$

(b) Solve the following :

$$\frac{dx}{dt} = 3x + 8y$$

$$\frac{dy}{dt} = -x - 3y$$

with $x(0) = 6$ and $y(0) = -2$.

(c) Solve : $x\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - \dfrac{dy}{dx} = 0.$

4. (a) Show that $J_1''(x) = -J_1(x) + \frac{1}{x}J_2(x)$.

(b) Prove that : $xP_{n-1}^1(x) + nP_{n-1}(x) = P_n^1(x)$.

(c) Evaluate $\int_{-1}^{+1} x^2 P_n^2(x)\,dx$.

5. (a) Find the Laplace transform of the following periodic function :

$$f(t) = \begin{cases} \sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

with period is $\frac{2\pi}{\omega}$.

(b) Find the inverse Laplace transform of :

(i) $\dfrac{s+1}{s^2-6s+25}$

(ii) $\dfrac{s}{\left(s^2+4\right)^2}$

(c) An alternative emf $E\sin\omega t$ is applied to circuit with an inductance L and a capacitance C in series. Show that the current in the circuit is :

$$\frac{E\omega}{\left(n^2-\omega^2\right)L}(\cos\omega t - \cos nt) \text{ where } n^2 = \frac{1}{LC}.$$

(a) Solve the partial differential equation $(x^2-yz)p+(y^2-zx)q=z^2-xy$.

(b) Solve $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial x\,\partial y}=\sin x\cos 2y$.

(c) Solve the partial differential equation $(D+1)(D+D^1-1)z=\sin(x+2y)$.

7. (a) Solve the following PDE by method of separation of variable

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial v}{\partial y}+2u, \text{ with } u(0,y)=0 \text{ and } \frac{\partial u(0,y)}{\partial x}=1+e^{-3y}.$$

(b) Find the temperature $u(x,t)$ in a slab whose ends $x=0$ and $x=l$ are kept at temperature zero and whose initial temperature $f(x)$ is given by

$$f(x)=\begin{cases} A & \text{when } 0<x<\dfrac{l}{2} \\ 0 & \text{when } \dfrac{l}{2}<x<l \end{cases}.$$

(c) Solve $\dfrac{\partial^2 v}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial v}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 v}{\partial \theta^2}=0$ with boundary conditions,

(i) v is finite when $r\to 0$

(ii) $v=\Sigma c_n \cos n\theta$ on $r=a$

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